

# Department of Mathematics & Statistics

Professor Jim Denier



# An overview of the Department



## SOME SIMPLE STATISTICS

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- Approximately 40 academic staff and 30 post-graduate students
- Staff awards, recognition and prizes
  - Fellow of the Australian Academy of Science – Emer. Prof. Ross Street
  - Australian Research Council Future Fellow – Dr Richard Garner
  - Australian Research Council DECRAAs – Dr Paul Bryan and Dr Chris Green
  - Australian Broadcasting Commission (ABC) *Top 5 Under 40* – Dr Sophie Calabretto
- Large taught program at undergraduate and postgraduate level
  - Bachelor of Mathematical Sciences (majors in: Applied Mathematics, Pure Mathematics and Statistics)
  - Master of Applied Statistics
  - Master of Data Science (joint with Department of Computing)
  - Around 10,000 students taught across undergraduate and postgraduate programs
- Rankings
  - Ranked in top 8 Australian Mathematical Sciences departments
  - Ranked at world standard in Applied Mathematics and well above world standard in Pure Mathematics

# Research in Applied Mathematics



- Research in Applied Mathematics
  - Rapidly growing research strength at Macquarie University
  - Significant strategic investment, underpinned by changes to undergraduate curriculum, driving research performance in this area
  - Unique mix of theory, computation and physical experiments
  - Major research themes spread across deterministic and stochastic modelling:

## **Deterministic Modelling:**

fluid mechanics, dynamical systems, pattern formation, microfluidics, wave propagation and scattering

## **Stochastic Modelling:**

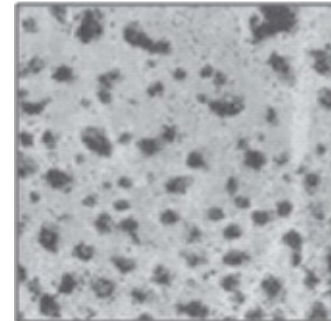
Bacterial growth and impact of crowding, optimisation and control

# Analysing patterns on curved surfaces

DR JUSTIN TZOU

In **harsh conditions**, vegetation may organise into **patterns** to conserve resources.

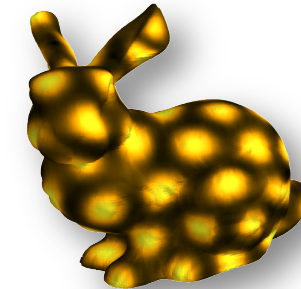
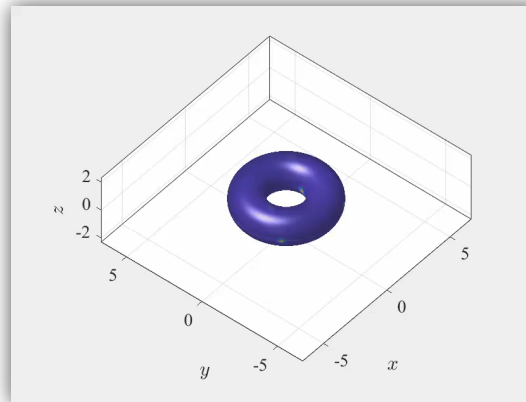
What determines the nature of the patterns?



We have shown **mathematically** that one determining factor is the **steepness** of the terrain.

A **long-standing unsolved problem** still remains: how does **curvature** of the terrain affect patterns?

We have developed a powerful **combination of pure and applied mathematical techniques** to answer this question

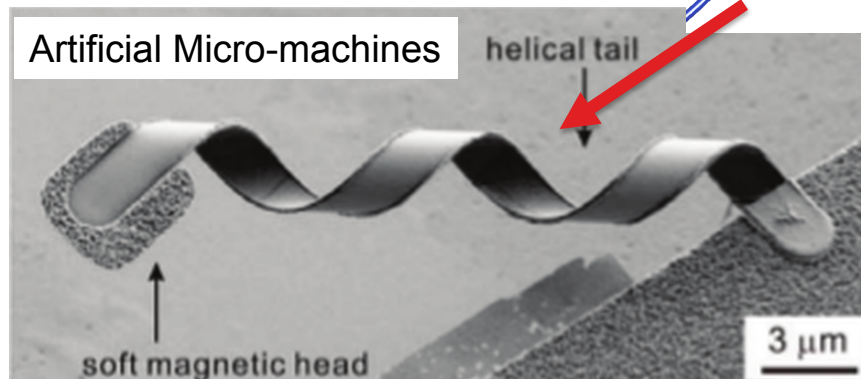
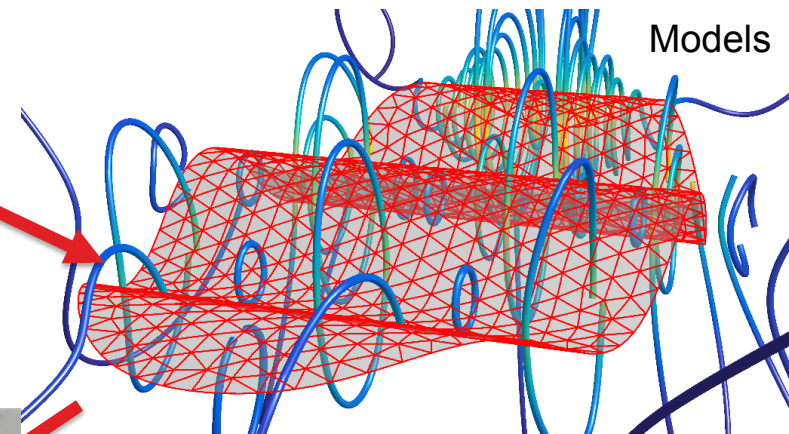
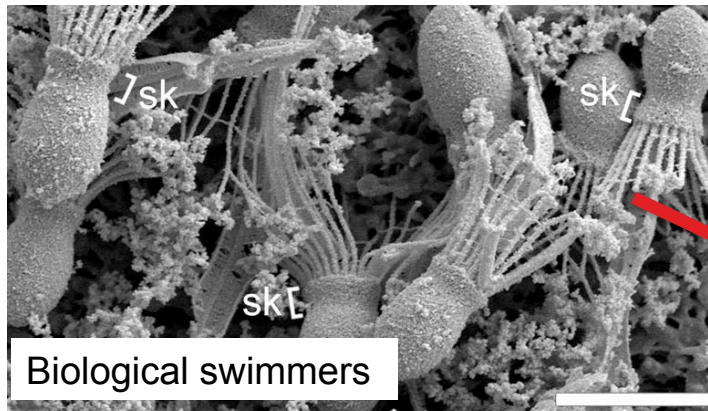


Other applications: animal markings (credit: C. Macdonald)



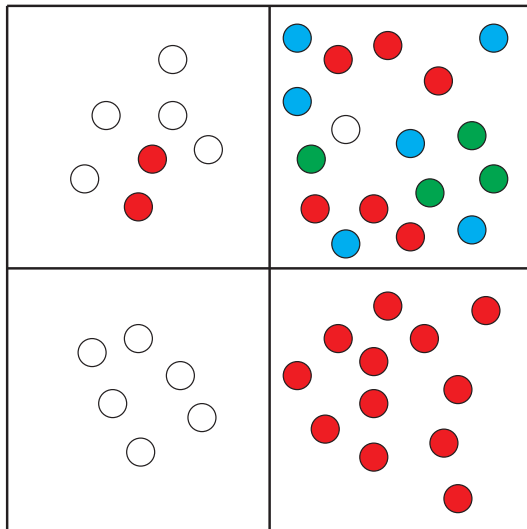
# Moving the Microscopic

DR LYNDON KOENS



# Blobs and Bacteria

DR CATHERINE PENINGTON



How fast does  
the box fill up  
with circles?

How many  
viruses can  
infect one  
bacterium?

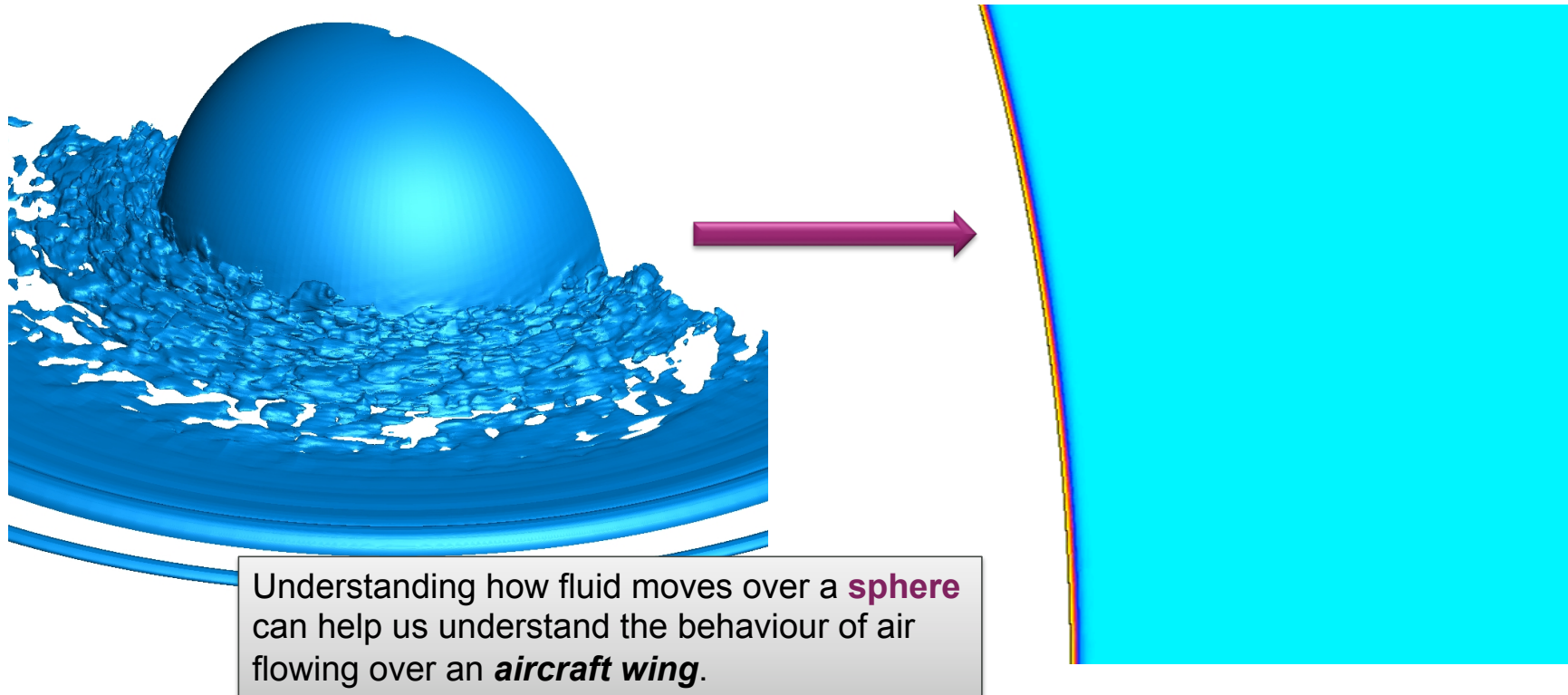
How fast do the  
bacteria spread out?



Studying how the blobs move and divide  
helps understand how biological cells  
move and divide

# Spinning spheres to make faster planes...?

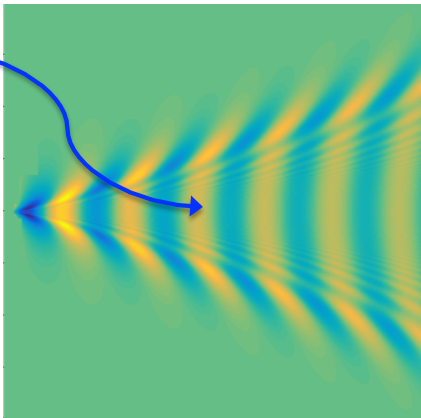
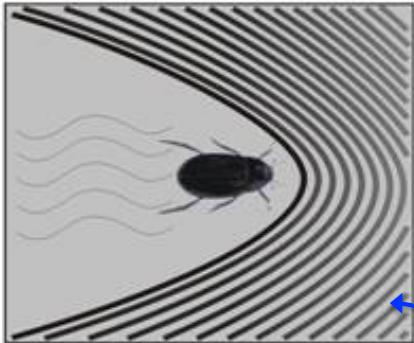
DR SOPHIE CALABRETTO



# When Do Small Things Have Big Effects?

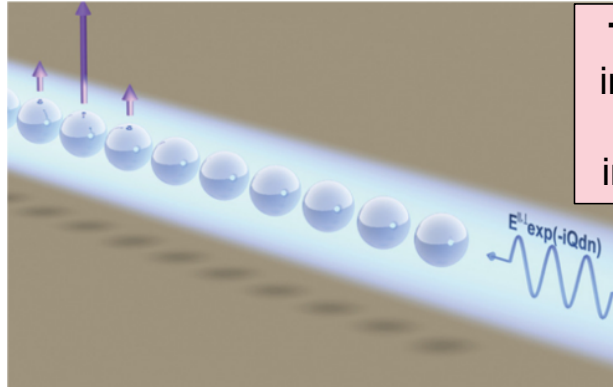
DR CHRISTOPHER LUSTRI

Gravity waves:  
caused by **gravity**  
pulling downwards



Capillary waves:  
caused by **surface tension** and curvature

Both gravity and surface tension effects are small, but the waves vanish entirely if we ignore them.



Tiny imperfections in particle chains can cause transmitted information to decay.

$E^i \exp(-iQdn)$

Can we understand combined gravity-capillary waves?

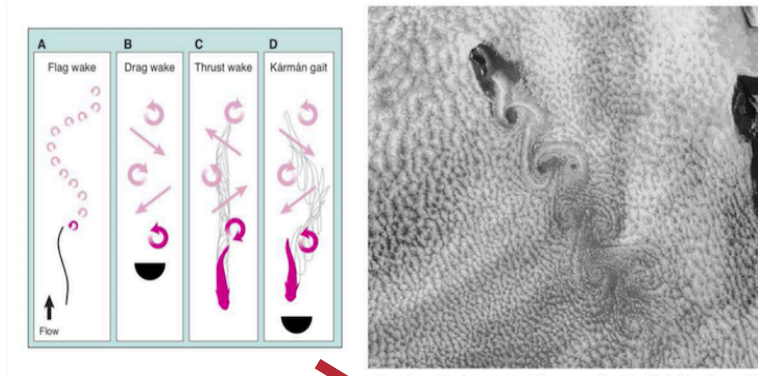
Can we reduce transmission decay in particle chains?

To answer these questions, we can *approximate* the solutions to uncover the roles played by the flow properties and chain imperfections.

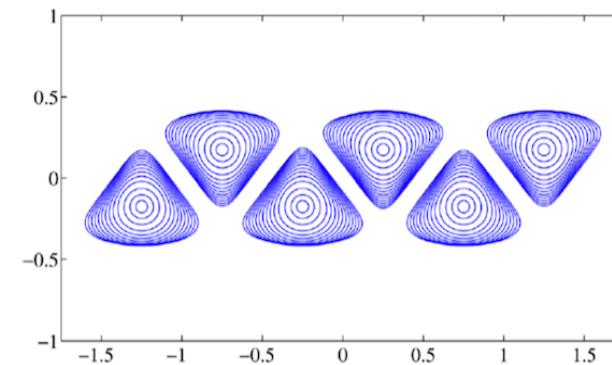
# Vortex dynamics and complex functions

DR CHRIS GREEN

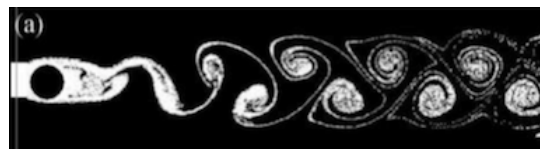
In nature, von Karman streets can be generated by fish and in cloud layers...



Complex functions can be used to construct a neat formula, which calculates the vortex shapes seen in nature!



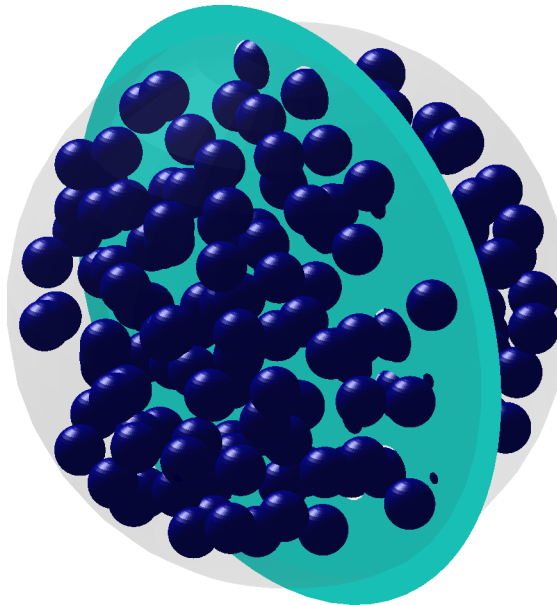
But what are the shapes of the vortices in a particular street?



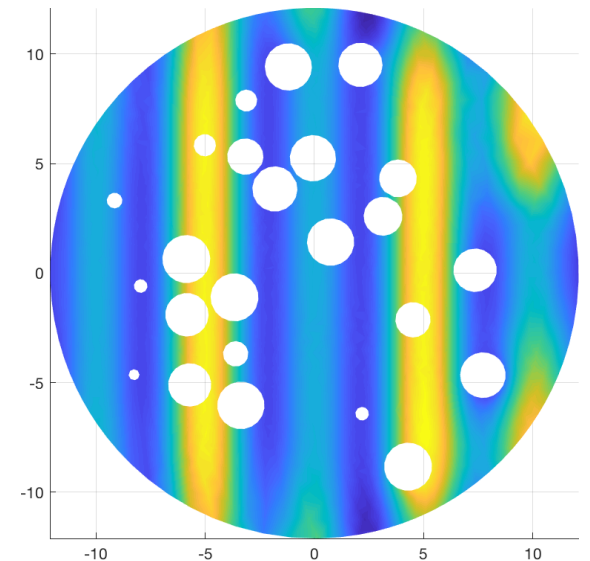


# Exa-scale multiple scattering simulations

DR STUART HAWKINS



State of the art algorithms to simulate wave scattering by clouds with hundreds of thousands of individual particles



# Instability in problems of control & optimisation

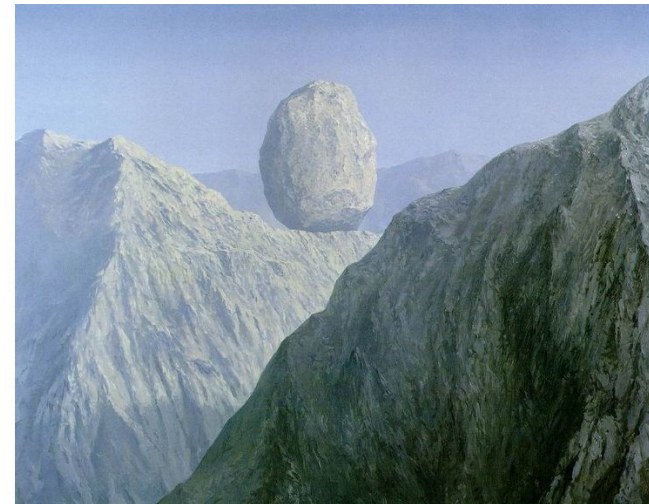
PROFESSOR VLAD GAITSGORY

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Problems of optimal control and optimisation that arise in applications are characterized by the fact that some parameters in their description are not known exactly or may change slowly in time.

In some problems, small changes of parameters may lead to significant and even dramatic changes in optimal or near optimal solutions.

An identification of cases when such an *instability* (or *oversensitivity*) with respect to small perturbations of parameters may occur and the development of mathematical tools to deal with it are of practical and theoretical importance.



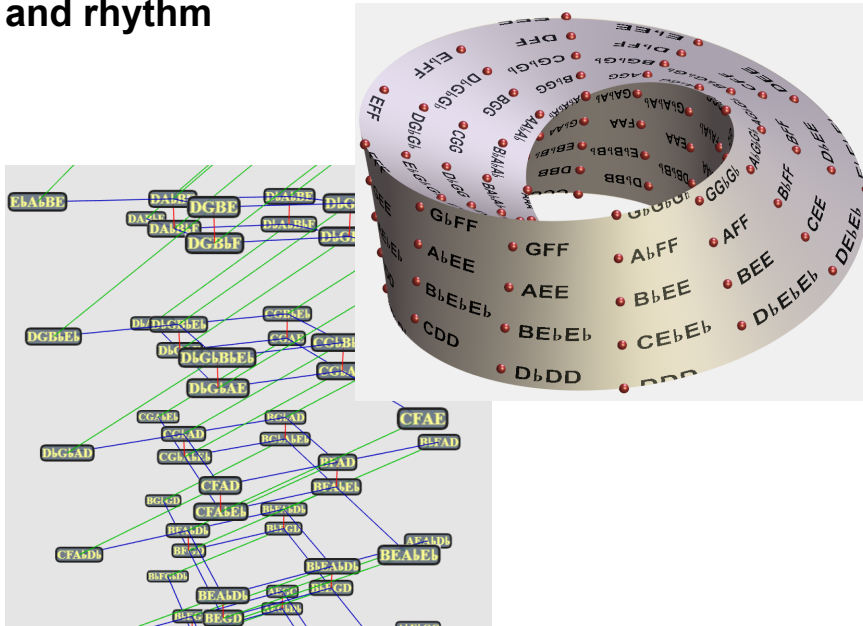


# Optimisation and music

DR DAVID BULGER

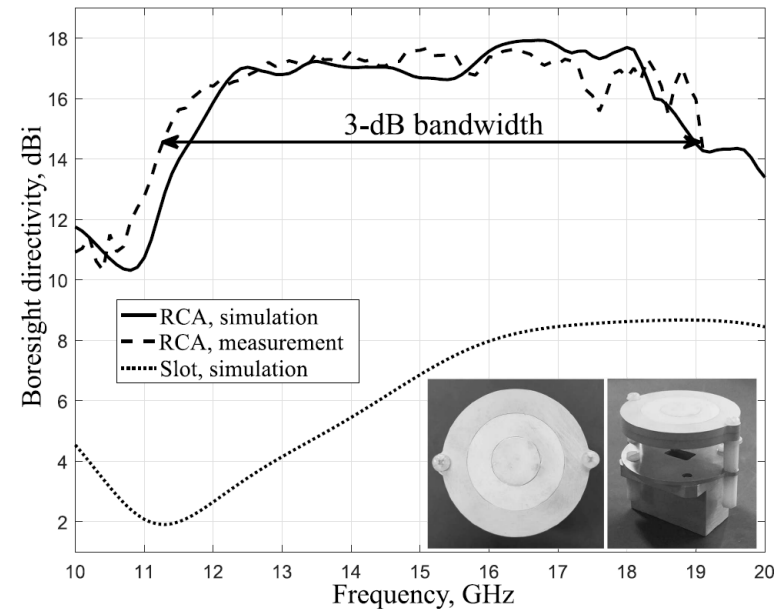
## Mathematical Musicology

Geometric and statistical models of harmony and rhythm



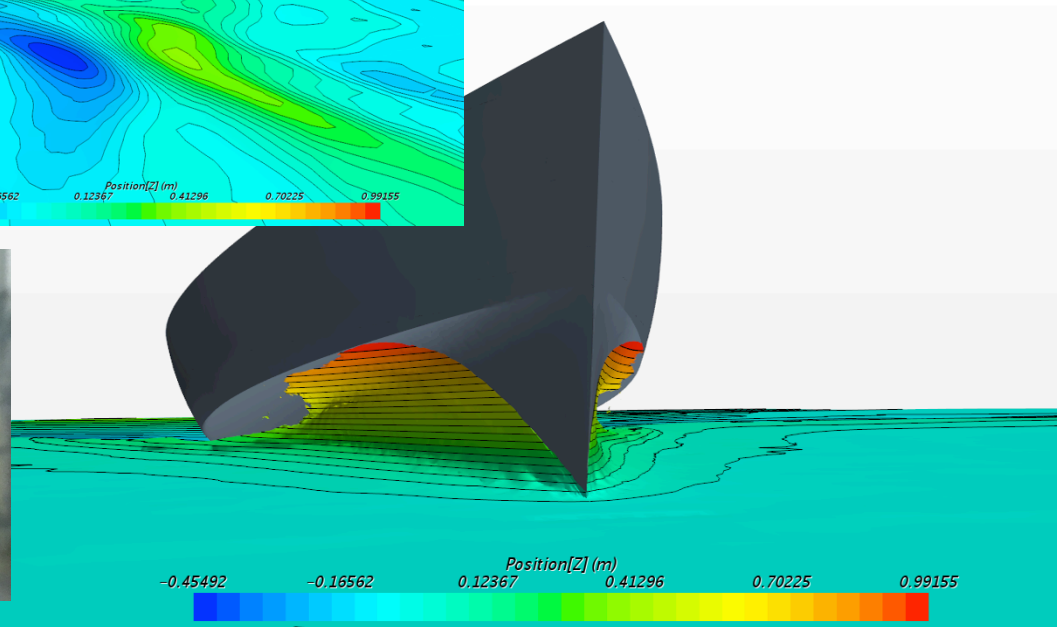
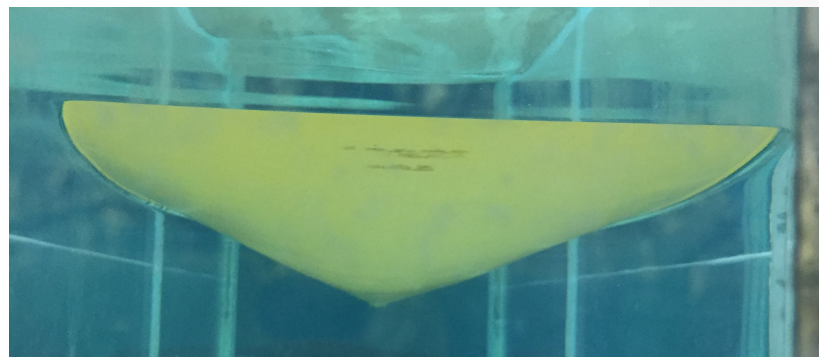
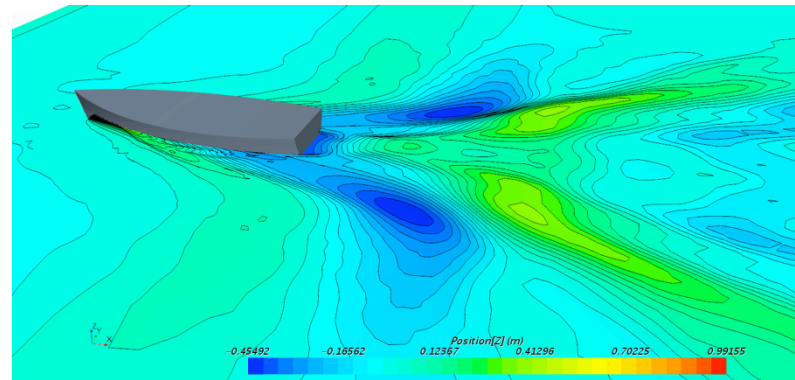
## Global optimisation

Algorithms for antenna design



# Minimising drag over boat-hulls

PROFESSOR JIM DENIER – HOD



# Research in Fundamental Statistics

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# Change-point detection

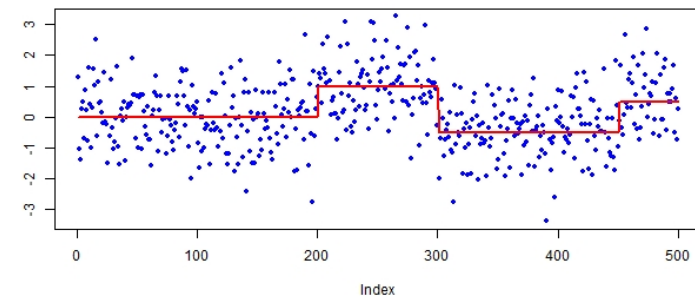
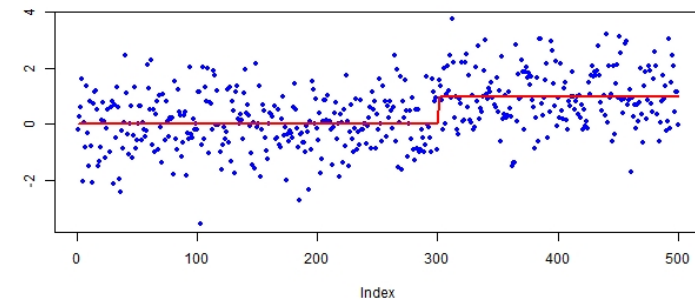
**DR GEORGY SOFRONOV – DIRECTOR OF RESEARCH**

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Change-point detection is an important problem arising in a wide range of fields: it is relevant to almost any experiment that generates a linear sequence of data. Its purpose is to identify points at which a property of interest changes.

Applications include

- Bioinformatics (DNA segmentation)
- Biomedical signal processing (structural analysis of EEG signals)
- Ecology (study of trends in abundance of various species)
- Economics (detection of structural changes)
- Epidemiology (timely detection and prevention of various types of diseases)
- Quality control (monitoring the production process and fault detection)
- Change-points in network data



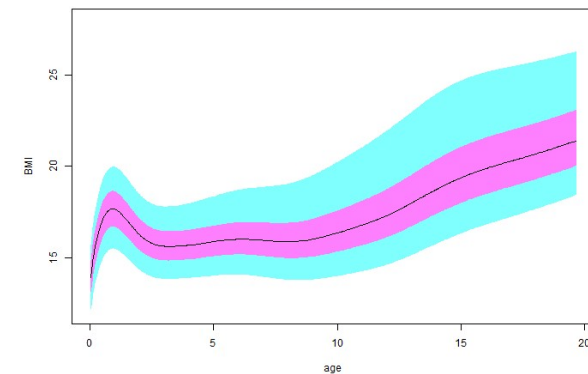
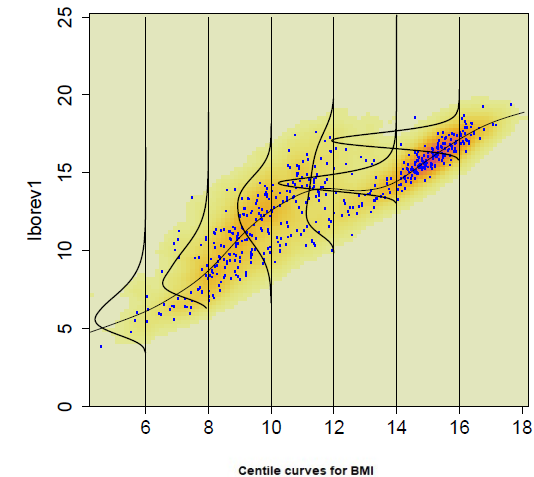
# Flexible regression modelling

PROFESSOR GILLIAN HELLER

*Flexible regression modelling* is a regression framework in which

- The response variable can have any distribution (as long as it's computable)
- Up to four distribution parameters can be modelled with covariates
- Approximately 100 distributions are available
- Smooth functions and random effects can be used
- Other features: zero-inflation, censoring, truncation, finite mixtures

Methodology is implemented in R package **gamlss** (available on CRAN)



# Penalized likelihood estimation in transformation models

DR MAURIZIO MANUGUERRA - DEPUTY HEAD OF DEPARTMENT

*Transformation models* are a general framework that can accommodate a wide family of models.

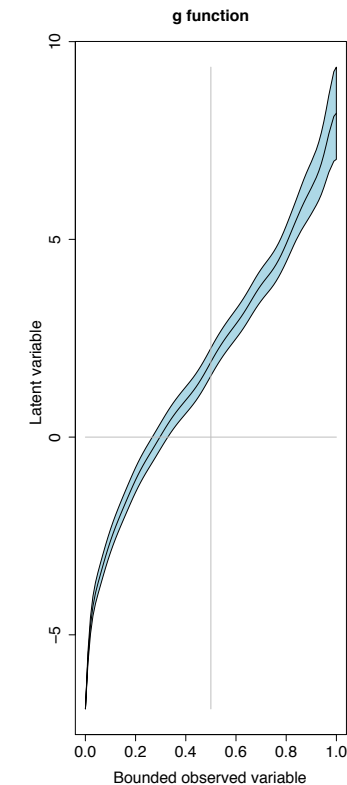
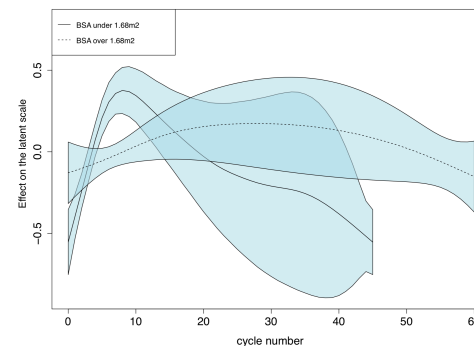
Maximum penalized likelihood methodology allows for the estimation of additive mixed models.

Modelling is performed on a latent scale which is linked to observations by a semi-parametric function estimated from data.

The methodology has been implemented in an R package available on CRAN: ordinalCont.

Applications include:

- ordinal regression for continuous scales
- survival analysis
- a variety of regression settings for bounded and unbounded scales.

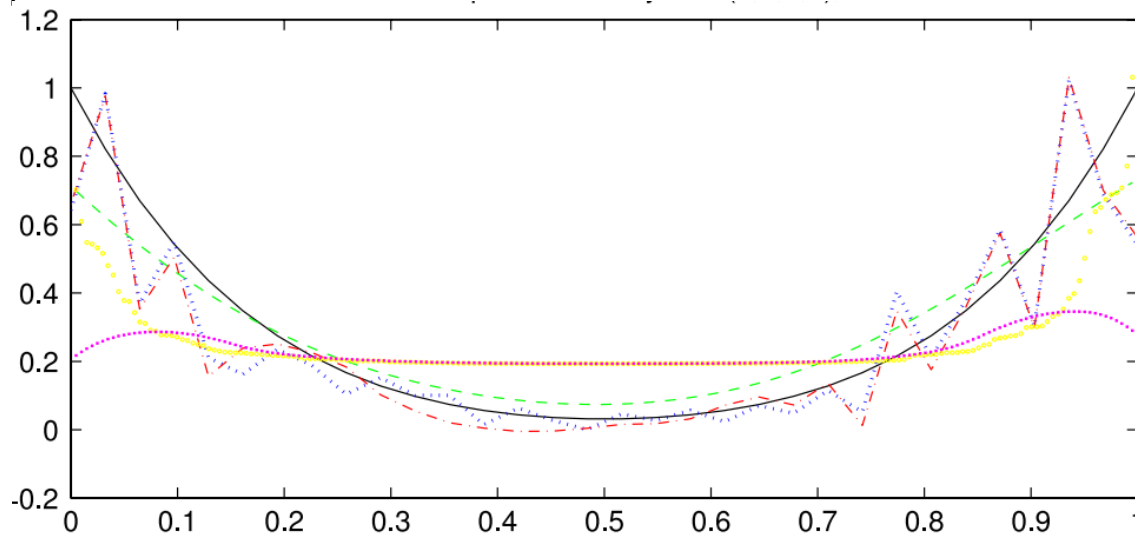


# Nonparametric Curve Estimation

DR HASSAN DOOSTI

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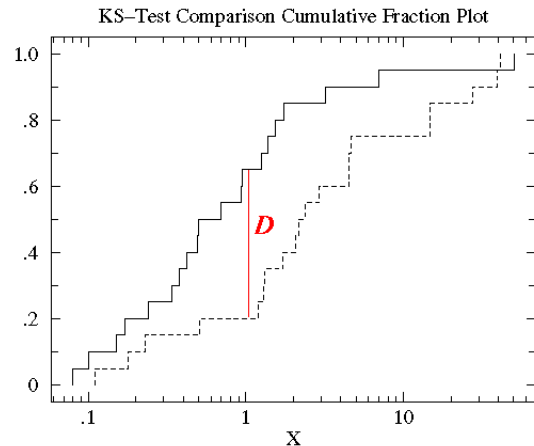
Estimation of an unknown function arises in many problems in Engineering, Medicine or Social Sciences. The estimation in a functional space often is dealt with using mathematical methods from Nonparametric Statistics. We continue development of new Nonparametric methods for estimation of unknown probability density functions, quantile functions, regression functions, intensity functions in multidimensional and spatial-temporal models.





# Testing for Difference: Weighted K-S Statistic

DR NINO KORDZAKHIA



Two sample Kolmogorov-Smirnov (K-S) statistic  $D$  is defined as

$$D = (1/m + 1/n)^{-1/2} \max_x |S_m(x) - S_n(x)|,$$

where  $S_m(x)$  and  $S_n(x)$  are empirical Cumulative Distribution Functions (CDF) for the samples of sizes  $m$  and  $n$ .

In short,  $D$  is the largest scaled difference between two ECDFs.

We consider the *weighted* K-S (*wK-S*) statistic  $D$  for specifically weighted samples selected, for example, from gene **A** (diseased) and **B** (control) sets in Gene Set Enrichment Analysis (GSEA). The evaluation of the asymptotic CDF of the *wK-S* reduces to computation of the distribution of the maximum of a *Gaussian* process, which is a modification of *Brownian bridge* process. In GSEA, the use of *wK-S* statistic leads to the discovery of statistically significant differences between **A** and **B**, which often appear to be a borderline case when tested using the two-sample K-S statistic.

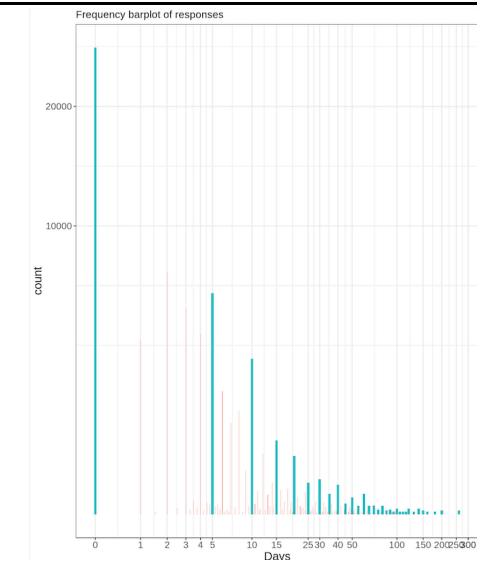
# Flexible Models for Time-Series of Counts

DR THOMAS FUNG

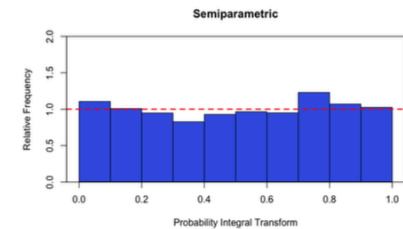
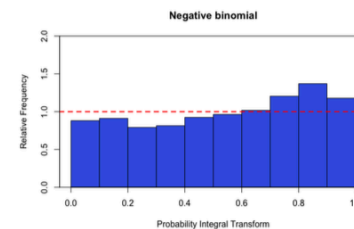
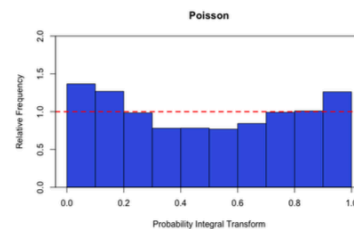
Time-series of counts are commonly observed in a range of fields as the collection of data often occurs over time.

Most methods in the literature assume that the postulated model for the data is correctly specified. In practice, however, real data typically exhibit deviations from an assumed model with problems such as overdispersion, zero inflation, truncation and heaping.

Incorrectly modelling such deviations leads to inefficient estimation and biased inferences on model parameters, resulting in invalid conclusions.



By adopting a semiparametric GLM framework, our model can automatically handle any deviations from parametric models.

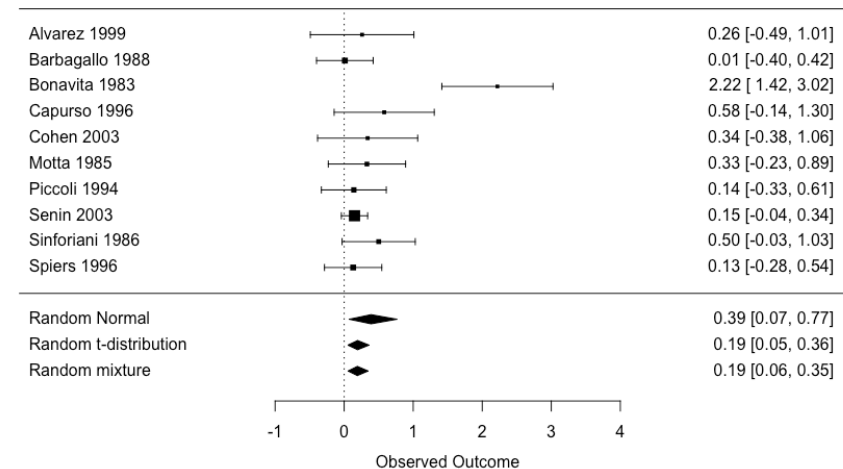


# Mixture models for robustness

DR KEN BEATH

- Assume that observations are a mixture of standard and outlier observations
- Use standard methods for model fitting, automatically adapts to characteristics of data and outliers
- Allows for robustness in models that may be otherwise difficult
- Posterior probabilities identify outliers
- Applied to meta-analysis and generalized linear models.
- Software, in the form of an R package, developed and made available.

## Robust meta-analysis



# Research in Pure Mathematics

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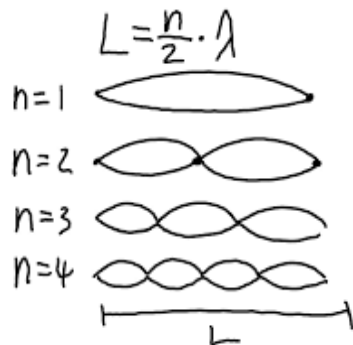


# Connection between heat and wave equations

**ASSOCIATE PROFESSOR ADAM SIKORA**

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The heat equation describes propagation of variation in temperature but it can also be used to model diffusion and Brownian motion.



The wave equation is a very important equation which can be used to model water waves, sound propagation, electromagnetic waves and many other phenomena. The simplest example is the vibration of a stretched elastic string



It is advantageous to study the interrelation between the heat and wave equations.

# Harmonic analysis and applications

**PROFESSOR XUAN DUONG, DR JI LI & DR THE ANH BUI**

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Harmonic analysis roots in the study of the decomposition of functions using Fourier series and the Fourier transform. Harmonic analysis has a central role in modern analysis with applications from vibrating strings to ocean tides to signal processing.

One of the major themes of harmonic analysis is the study of singular integrals on certain function spaces which relies on the theory of Calderon-Zygmund operators. However, in practice there are a number of important settings which do not fall within the scope of the Calderon-Zygmund theory. My research develops the theory of singular integrals on new function spaces beyond the Calderon-Zygmund theory and investigates applications to partial differential equations.



# Monoidal categories model open systems

ASSOCIATE PROFESSOR STEVE LACK

Monoidal categories arise in many areas of mathematics, including algebra, geometry, and logic.

They can also be used to model systems which can be built up out of subsystems via series and parallel composition.

This can be used to assist in the modular design of systems.

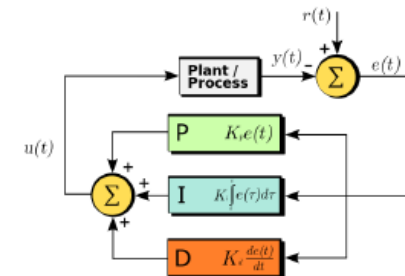
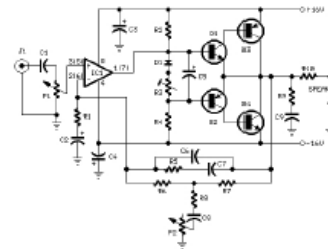
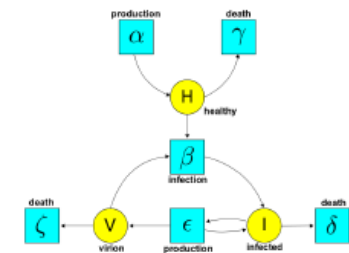
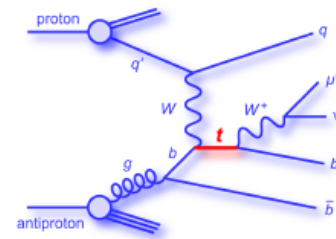


Image: John Baez



# Self-similarity and category theory

DR RICHARD GARNER

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Self-similarity pervades mathematics. A basic example involves the equation

$$X = X + X$$

and its solutions in different domains of discourse:

- In topology, it gives rise to *Cantor space*:



- In algebra, it gives rise to *Leavitt algebras*.
- In analysis, it gives rise to *Cuntz C\*-algebras*.

In recent research, I have sought to link these structures via the tools of *category theory*.

Category theory allows one to describe self-contained mathematical universes, each adhering to their own laws and logical principles.

I have considered a particular mathematical universe which is inherently “self-similar”, and within which structures like those listed above appear naturally.

I am currently expanding this line of enquiry to incorporate other self-similar structures such as *Leavitt path algebras* and *Cuntz-Kreiger C\*-algebras*.

Thank you